

Hypertriton Electric Polarizability

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A rigorous formalism for determining the electric dipole polarizability of a three-hadron bound complex in the case that the system has only one bound (ground) state has been elaborated. On its basis, by applying a model wave function that takes into account specific features of the structure of the lightest hypernucleus and using the known low-energy experimental data for the $p - n$ and $\Lambda - d$ systems as input data, we have calculated the value of the electric dipole polarizability of the lambda hypertriton $\alpha_E(^3\Lambda\text{H})$. It follows from our study that the polarizability of the lambda hypertriton is close to 3 fm^3 exceeding the polarizabilities of the ordinary three-nucleon nuclei by an order of magnitude.

Keywords: Electric dipole polarizability; lambda hypertriton.

PACS Nos.: 21.10.Ky; 21.45.+v; 21.80.+a

1. Introduction

Data on the polarization (deformation) of the few-body nuclear systems in the electromagnetic field that is a fundamental property of each nucleus contain important information on the nuclear force. To date, by studying deviations from Rutherford scattering below the Coulomb barrier of a light nucleus in an intense Coulomb field of a heavy nucleus, the values of the electric dipole polarizability of the two lightest nuclei, of the deuteron and the ^3He nucleus, have been directly measured,

$$\alpha_E(^2\text{H}) = 0.70 \pm 0.05 \text{ fm}^3 \text{ (Ref. 1)}, \quad \alpha_E(^3\text{He}) = 0.25 \pm 0.04 \text{ fm}^3 \text{ (Ref. 2)}. \quad (1)$$

An other way of obtaining the value α_E , that is based on the known relation between the polarizability and an energy-weighted integral over the total photoabsorption cross-section for the corresponding nucleus σ (the sum rule σ_{-2}) with the use of the experimental data for σ , leads to the values $\alpha_E(^2\text{H}) = 0.61 \pm 0.04 \text{ fm}^3$ (Ref. 3) and $\alpha_E(^3\text{He})$ in the range from 0.13 to 0.17 fm^3 (Ref. 2), supporting the result of the direct measurement in the case of the deuteron but being inconsistent with that in the case of the ^3He nucleus.

Calculations of the deuteron electric polarizability, carried out in the framework of the traditional nuclear physics for the realistic nucleon-nucleon interaction potentials, lead to the values of $\alpha_E(^2\text{H})$ in the range from 0.631 to 0.634 fm^3 (Refs. 4 and 5), being consistent with experiment. Furthermore, examining theoretically the anisotropy of the deformation of the deuteron in the electric field caused by the tensor character of the $n - p$ interaction^{5,6} each of the components of the deuteron electric polarizability $\alpha_E^{[M]}$, the longitudinal component (with the deuteron spin along

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the electric field) α_E^1 and the transverse one α_E^0 , have been individually calculated,

$$\alpha_E^1(^2\text{H}) = 0.669 \text{ fm}^3, \text{ and } \alpha_E^0(^2\text{H}) = 0.555 \text{ fm}^3 \text{ (Ref. 5)}. \quad (2)$$

(The above electric polarizability $\alpha_E(^2\text{H})$ is the averaged value of the components $\alpha_E^{|M|}$, $\alpha_E(^2\text{H}) = \frac{2}{3}\alpha_E^1 + \frac{1}{3}\alpha_E^0$.) The similar components of the paramagnetic dipole susceptibility of the deuteron have been calculated in Ref. 7.

Computations of scalar and tensor deuteron polarizabilities have also been performed in the framework of the effective field theory that uses space-time and global chiral symmetries of the quantum chromodynamics consistently describing pion propagation and relativistic effects^{8,9}. Though the results for the electric and magnetic deuteron polarizabilities obtained in the cited works in the leading and next-to-leading orders agree with the values calculated in the traditional nuclear physics with the application of the potential models, it can look forward the occurrence of deviations from the values at higher orders in the effective field theory expansion.

Presently, few-body methods are widely used in the physics of hypernuclei — hadronically stable bound formations of protons, neutrons and one or more hyperons. Data on properties of few-hadron systems are the basic source of obtaining information on the interaction forces both between Λ hyperon and nucleon ($\Lambda - N$ interaction) and between two Λ hyperons ($\Lambda - \Lambda$ interaction). To date, several tens lambda hypernuclei have been found experimentally. The very lightest hypernuclei are a three-hadronic system consisting of the proton, the neutron and the Λ hyperon in a bound state, the hypertriton ${}^3_\Lambda\text{H}$, and the two mirror four-hadron nuclei, ${}^4_\Lambda\text{H}$ and ${}^4_\Lambda\text{He}$, that are bound $pnn\Lambda$ and $ppn\Lambda$ systems, respectively. The hypertriton is the simplest halo nucleus, moreover, with the unique strange halo. It can be considered as a weakly bound two-body formation from the Λ particle and a core in the form of the deuteron which are separated by distances substantially exceeding the deuteron size.

Of four double- Λ hypernuclei reported thus far the lightest are ${}^4_{\Lambda\Lambda}\text{H}$ (Ref. 10) and ${}^6_{\Lambda\Lambda}\text{He}$ (Refs. 11 and 12). Evidence on the production of the hypernucleus ${}^4_{\Lambda\Lambda}\text{H}$ in (K^-, K^+) on ${}^9\text{Be}$ has been presented after analysing the experiment E906 completed at the Alternating Gradient Synchrotron of the Brookhaven National Laboratory in Ref. 10. However, to date there arises a doubt as to existence of the bound system ${}^4_{\Lambda\Lambda}\text{H}$, since its formation and decay were not needed to explain the data of this experiment¹³. Hence, the existence of hypernucleus ${}^4_{\Lambda\Lambda}\text{H}$ has not been conclusively proved yet.

This paper is devoted to study of the behaviour of the lightest of all Λ hypernuclei, the hypertriton ${}^3_\Lambda\text{H}$, in the external Coulomb field. A new expression for the electric dipole polarizability of the three-hadron bound complex, obtained by us in the special case that the system has only one bound (ground) state, is given in Section 2. The application of this expression to the hypertriton in the framework of a physically justified model is considered in Section 3. The results of corresponding calculations of $\alpha_E(^3_\Lambda\text{H})$ based on the known low-energy data for the $p - n$ and $\Lambda - d$ interactions are discussed in Section 4. Section 5 is devoted to a short summary and conclusions.

2. Electric polarizability of the three-hadron bound system

We derive an expression for the electric dipole polarizability of a bound hadronic complex (that consists of N particles), α_E , on the basis of the rigorous few-body approach using the formalism of the effective interaction of the charged complex with external Coulomb field, the source of which can be any charged particle or nucleus (the particle 1). The electric dipole polarizability of the complex characterizes the strength of the effective potential of interaction between the particle 1 and the centre of mass of particles of the complex at asymptotically large distances. In the simplest event that the complex contains only one charged particle (the particle 2) and the rest are neutral, the electric dipole polarizability of the complex can be written as

$$\alpha_E = -2 \langle \Psi_0 | (\mathbf{D}_2 \cdot \hat{\boldsymbol{\rho}}_1) G^Q(-B_0) (\mathbf{D}_2 \cdot \hat{\boldsymbol{\rho}}_1) | \Psi_0 \rangle, \quad (3)$$

where Ψ_0 , $\mathbf{D}_2 = e_2 \mathbf{r}_2$, and $G^Q(-B_0)$ are the wave function of the ground bound state of the complex (of N particles) corresponding to the binding energy B_0 (normalized to unit, $\langle \Psi_0 | \Psi_0 \rangle = 1$), the operator of the dipole moment of the charged particle 2 (having the charge e_2) of the complex, and the product of the projection operator $Q = 1 - P$ (P is the projection operator on to the complex ground state, $P = |\Psi_0\rangle\langle\Psi_0|$) and the full Green's operator of the complex $G(E) = (E - H_0 - V)^{-1}$, $G^Q(E) = QG(E)$, at the energy $E = -B_0$, respectively. Here H_0 is the kinetic energy operator of the complex and V is the total interaction potential. The latter is given as $V = \sum_{i<j} v_{ij}$ where v_{ij} is the potential of the pair interaction between the particles i and j , the summation is over all the particles of the complex 2, 3, ..., N . The quantities $\boldsymbol{\rho}_1$ and \mathbf{r}_2 in Eq. (3) are the radius vectors specifying the relative positions of the particle 1 forming the external Coulomb field and the particle 2, the charged constituent of the complex, both with respect to the centre of mass of the complex (the hat signifies a unit vector: $\hat{\boldsymbol{\rho}}_1 \equiv \boldsymbol{\rho}_1/\rho_1$).

Previously, on the basis of the three-body formalism of the effective interaction of a charged particle and a bound complex¹⁴⁻¹⁷, we have derived an expression for the polarization potential of the two-hadron ($N = 2$) bound complex that consists of charged and neutral hadrons (for example, the deuteron)^{15,18,19}, starting immediately from the Faddeev integral equations²⁰. In the case that the interaction between the proton (the particle 2) and the neutron composed the deuteron is central, the general formula (3) for the electric dipole polarizability of the deuteron is reduced to

$$\alpha_E(^2\text{H}) = \frac{2}{3} \frac{e^2}{\hbar^2 c^2} \left(\frac{m_n}{m_{pn}} \right)^2 \int_0^\infty \frac{dk k^2}{2\pi^2} \frac{|\psi'_0(k)|^2}{\frac{k^2}{2\mu_{pn}} + B_d}, \quad (4)$$

where e is the charge of the proton, μ_{pn} is the proton-neutron reduced mass, $\mu_{pn} = m_p m_n / m_{pn}$, $m_{pn} = m_p + m_n$ (m_p and m_n are the proton and neutron masses), $\psi'_0(k) \equiv d\psi_0(k)/dk$ is the first derivative of the deuteron wave function in the momentum space in the variable of magnitude of the relative momentum k and B_d is the binding energy of the deuteron. The formula (4) is in agreement with the expression for α_E obtained in the case of the separable S-wave pair potential in Refs. 15, 18 and 19.

In this work, to derive the formula for the electric dipole polarizability of the hypertriton ${}^3_\Lambda\text{H}$ ($N = 3$), we start from the four-body problem, considering the bound system of the charged particle — the proton (the particle 2) — and two neutral particles — the neutron (the particle 3) and the Λ hyperon (the particle 4) — in the Coulomb field of the charged particle 1. Taking into consideration the analytical properties of the three-body transition matrix that determines the Green's function in Eq. (3), we find the following expression for the electric dipole polarizability of the hypertriton:

$$\alpha_E({}^3_\Lambda\text{H}) = 2 \frac{e^2}{\hbar^2 c^2} \int \frac{d\mathbf{k} d\mathbf{p}}{(2\pi)^6} \frac{\left| \left(\frac{m_n}{m_{pn}} \nabla_{\mathbf{k}} + \frac{m_\Lambda}{m_{pn\Lambda}} \nabla_{\mathbf{p}} \right) \Psi_0(\mathbf{k}, \mathbf{p}) \right|^2}{\frac{k^2}{2\mu_{pn}} + \frac{p^2}{2\mu_{pn,\Lambda}} + B_{ht}}, \quad (5)$$

where $\mu_{pn,\Lambda}$ is the reduced mass of the proton-neutron system with the mass $m_{pn} = m_p + m_n$ and the Λ hyperon with the mass m_Λ , $\mu_{pn,\Lambda} = m_{pn}m_\Lambda/m_{pn\Lambda}$, $m_{pn\Lambda} = m_p + m_n + m_\Lambda$, $B_{ht} = B_d + B_\Lambda$ is the binding energy of the hypertriton that equals to the sum of the deuteron binding energy $B_d = \kappa_d^2/2\mu_{pn}$ and the binding energy of the Λ hyperon $B_\Lambda = \kappa_\Lambda^2/2\mu_{d,\Lambda}$, $\mu_{d,\Lambda} = m_d m_\Lambda/m_{d\Lambda}$, m_d is the mass of the deuteron, $m_{d\Lambda} = m_d + m_\Lambda$, $\Psi_0(\mathbf{k}, \mathbf{p})$ is the normalized wave function of the hypertriton in the momentum space, \mathbf{k} and \mathbf{p} being the Jacobi momentum variables describing the relative motion of the proton p and the neutron n and the motion of the centre of mass of p and n relative to the particle Λ , respectively

$$\mathbf{k} = \frac{m_n \mathbf{k}_p - m_p \mathbf{k}_n}{m_{pn}}, \quad \mathbf{p} = \frac{m_\Lambda (\mathbf{k}_p + \mathbf{k}_n) - m_{pn} \mathbf{k}_\Lambda}{m_{pn\Lambda}}, \quad (6)$$

k_i is the momentum of the particle i .

We emphasize that the formula for the electric polarizability of the three-hadron system (5) has been derived on the assumption that the interactions between constituents support the existence of only one (ground) bound state of the system. As this takes place, the formula (5) provides a rigorous treatment of the problem. It is significant that the approach developed by us does not require a knowledge of the three-body wave functions of the continuum states.

3. Wave function of the hypertriton

Unlike the wave function of the triton, the wave function of the hypertriton has a distinct cluster character. The $\Lambda - N$ interaction force is weaker than the $n - p$ one to an extent that each of the interaction potentials $v_{\Lambda p}$ and $v_{\Lambda n}$ by itself does not form a bound state — the hyperdeuteron does not exist. Moreover, in the hypertriton the Λ hyperon is bound up with both nucleons much weaker than the proton and the neutron are bound up together in the deuteron ($B_\Lambda/B_d = 0.058$). The deuteron in the hypertriton is only slightly deformed by the Λ hyperon. Therefore, the hypertriton may be considered as a two-cluster bound system composed of the Λ hyperon and a core in the form of the deuteron. Herewith, the hypertriton size determined by the distance between the Λ hyperon and the deuteron centre of mass substantially exceeds the deuteron size. In accordance with the uncertainty principle, it follows that in the hypertriton the motion of the Λ hyperon relative to the centre of mass

of the deuteron proceeds slowly in comparison to a quicker relative motion of the nucleons p and n inside the deuteron: $p/k \propto \kappa_\Lambda/\kappa_d = 0.295$.

Under these conditions, it may be approximately considered that in the hypertriton the relative proton-neutron motion and the relative motion of the Λ hyperon and the deuteron centre of mass operates independently of one another. Then the wave function of the hypertriton is factorized taking the form

$$\Psi_0(\mathbf{k}, \mathbf{p}) = \psi_0(k)\phi_\Lambda(p), \quad (7)$$

where $\psi_0(k)$ is the normalized wave function of the deuteron in the ground state with the binding energy B_d corresponding to the pair interaction potential v_{pn} , and $\phi_\Lambda(p)$ is the normalized wave function of the ground bound state of the Λ hyperon and the deuteron with the binding energy B_Λ that corresponds to the effective $\Lambda - d$ interaction potential v_Λ . The potential v_Λ depends only on the radius-vector of the particle Λ relative to the centre of mass of the deuteron \mathbf{p} (or on the variable momentum \mathbf{p} in the momentum space).

The expression (7) can be considered as the first term of an expansion of the three-particle function Ψ_0 in terms of the complete set of wave functions of the complex of the interacting proton and neutron that corresponds to the ground bound state of the complex. In Eq. (7) all the summands of the expansion in terms of the functions of the continuous spectrum of the complex $p + n$ were neglected.

The dominance of the factorized term (7) in the hypertriton wave function has been substantiated rigorously by the three-body calculations with the use of the realistic potentials in Ref. 21. According to them, the integral of the square of the overlap of the deuteron with the hypertriton normalized wave functions $\phi_\Lambda(p) \equiv \langle \psi_0 | \Psi_0 \rangle$ taken over all values of the variable that describes the relative motion of Λ and the centre of mass of the deuteron, $P_d({}^3_\Lambda\text{H}) \equiv \langle \phi_\Lambda | \phi_\Lambda \rangle$, is found to be $P_d({}^3_\Lambda\text{H}) = 0.987$. From this result, close to unity ensured by the model (7), the cluster character of the hypertriton wave function is immediately evident. For comparison, in the case of the triton the integral of the square of the overlap of the deuteron with the triton wave functions is equal to²¹ $P_d({}^3\text{H}) = 0.445$ that indicates inadequacy of the simplest cluster model (7) for description of the triton, more sophisticated models are needed in such a case.

In our work, to evaluate the magnitude of the electric dipole polarizability of the hypertriton, we use the function Ψ_0 in the form (7) accounting for the above-mentioned peculiarity of the hypertriton structure with a high probability of finding two nucleons in the state of the deuteron. Substituting (7) into the rigorous three-particle formula (5) for $\alpha_E({}^3_\Lambda\text{H})$, the expression (5) takes the form

$$\alpha_E({}^3_\Lambda\text{H}) = \frac{4}{3} \frac{e^2}{\hbar^2} \frac{m_p m_n}{m_{pn}} \int_0^\infty \frac{p^2 dp}{2\pi^2} \left\{ \left(\frac{m_\Lambda}{m_{pn\Lambda}} \right)^2 I(p) [\phi'_\Lambda(p)]^2 + \left(\frac{m_n}{m_{pn}} \right)^2 J(p) [\phi_\Lambda(p)]^2 \right\}, \quad (8)$$

where the functions $I(p)$ and $J(p)$ are defined by

$$I(p) = \int_0^\infty \frac{dk k^2}{2\pi^2} \frac{[\psi_0(k)]^2}{k^2 + [C(p)]^2}, \quad J(p) = \int_0^\infty \frac{dk k^2}{2\pi^2} \frac{[\psi'_0(k)]^2}{k^2 + [C(p)]^2}, \quad (9)$$

$$[C(p)]^2 \equiv \frac{m_p m_n}{m_{pn} m_\Lambda} \left\{ \frac{m_{pn\Lambda}}{m_{pn}} p^2 + \frac{m_{d\Lambda}}{m_d} \kappa_\Lambda^2 \right\} + \kappa_d^2 .$$

The wave functions $\psi_0(k)$ and $\phi_\Lambda(p)$ in (8) and (9) were determined by solving the corresponding two-body problems with S-wave separable interaction potentials — for the system proton + neutron with the interaction potential v_{pn} ,

$$v_{pn}(k, k') = -\gamma_{pn} w_{pn}(k) w_{pn}(k') \quad (10)$$

and for the Λ hyperon in a field of the effective interaction potential v_Λ ,

$$v_\Lambda(p, p') = -\gamma_\Lambda w_\Lambda(p) w_\Lambda(p'), \quad (11)$$

where the parameters γ_{pn} and γ_Λ characterize the strength of the corresponding interaction and the formfactors $w_{pn}(k)$ and $w_\Lambda(p)$ describe dependences of the interaction potentials on the variable momenta k and p . We use the $p - n$ and $\Lambda - d$ interaction potentials, (10) and (11), with the formfactors of three different kinds — Yukawa formfactor (Yu), exponential (in configuration space) formfactor (Exp) and the formfactor corresponding to the delta-shell potential (DS),

$$w^{\text{Yu}}(k) = \frac{\beta^2}{k^2 + \beta^2}, \quad w^{\text{Exp}}(k) = \frac{\beta^4}{(k^2 + \beta^2)^2}, \quad w^{\text{DS}}(k) = \frac{\sin(k/\beta)}{(k/\beta)}. \quad (12)$$

All the formfactors are normalized to one at zero momentum, $w(0) = 1$.

With a certain formfactor, each of the potentials (v_{pn} and v_Λ) is given by two parameters which characterize the strength and the inverse radius of interaction (γ_{pn}, β_{pn} and $\gamma_\Lambda, \beta_\Lambda$). The model of zero-range interaction (ZR) follows immediately from the model of separable potential with any one of the formfactors (12) providing the parameter of the inverse radius of interaction takes an infinite large value: $\beta \rightarrow \infty$.

4. Calculations and discussion of results

Calculations of the electric dipole polarizability of the hypertriton $\alpha_E(^3\Lambda\text{H})$ were carried out by the formulae (8) and (9). The two-body wave functions $\psi_0(k)$ and $\phi_\Lambda(p)$ were found using both the model of zero-range interaction (ZR) and the model of the separable S-wave interaction (10) and (11) with the different formfactors (12).

The parameters of the $p - n$ interaction potentials were fitted to the experimental values of the deuteron binding energy B_d (Ref. 22) and the triplet $p - n$ scattering length $^3a_{pn}$ (Ref. 23),

$$B_d = 2.224575(9) \text{ MeV} \quad (\text{Ref. 22}), \quad ^3a_{pn} = 5.424(3) \text{ fm} \quad (\text{Ref. 23}). \quad (13)$$

Of the parameters needed for fixing the low-energy effective $\Lambda - d$ interaction potential (11) with the formfactors (12), it is known to date only one — the binding energy of the Λ hyperon in the hypertriton B_Λ ,

$$B_\Lambda = 0.13(5) \text{ MeV} \quad (\text{Ref. 24}). \quad (14)$$

Relying on just one parameter — on the experimental value of the binding energy B_Λ — the $\Lambda - d$ interaction can be described only at very low energies by applying the zero range interaction model.

To evaluate the influence of finiteness of the effective $\Lambda - d$ interaction radius on the magnitude of the polarizability $\alpha_E(^3\Lambda\text{H})$ it is worthy to utilize as an second parameter the theoretical value of the doublet Λ hyperon - deuteron scattering length $^2a_{\Lambda d}$ calculated by a number of authors on the basis of the three-particle description of the hypertriton as a bound system of the proton, the neutron and the Λ hyperon with the use of the low-energy data for $p - n$ and Λ -nucleon interactions (see Ref. 25 and citations therein). The results of analysis in Refs. 25 and 26 point to the existence of a simple correlation dependence between B_Λ and $^2a_{\Lambda d}$ that follows readily from the effective range theory of the $\Lambda - d$ interaction under condition that the B_Λ is much smaller than the deuteron binding energy B_d . According to the correlation, to the experimental value B_Λ (14) there corresponds²⁵ the doublet scattering length

$$^2a_{\Lambda d} = 15.9 \text{ fm} . \quad (15)$$

Notice that the value (15) is consistent with the values calculated recently in the framework of the effective field theory in Ref. 27.

The fitted values of the parameters of the interaction potentials v_{pn} and v_Λ (γ_{pn}, β_{pn} and $\gamma_\Lambda, \beta_\Lambda$) are presented in the Table 1 for the various formfactors w_{pn} and w_Λ (12).

Table 1. The parameters of the potentials v_{pn} and v_Λ , (8) and (9), with the formfactors (12) that correspond to the experimental values of the deuteron binding energy B_d and the triplet $p - n$ scattering length $^3a_{np}$ (13) (for $p - n$ interaction) and to the experimental binding energy of the Λ hyperon in the hypertriton B_Λ (14) and the theoretically evaluated value of the doublet $\Lambda - d$ scattering length $^2a_{\Lambda d} = 15.9 \text{ fm}^{25}$ (for $\Lambda - d$ interaction).

Formfactor	System $p + n$		System $\Lambda + d$	
	$2\mu_{pn}\gamma_{pn}, \text{fm}$	$\beta_{pn}, \text{fm}^{-1}$	$2\mu_{\Lambda d}\gamma_\Lambda, \text{fm}$	$\beta_\Lambda, \text{fm}^{-1}$
Yu	24.5955	1.3906	23.5403	1.1934
Exp	27.5011	2.0522	25.9893	1.7485
DS	27.7483	0.6372	26.5333	0.5365

In the case of using the zero-range model for both interactions (between the proton and the neutron and between the Λ hyperon and the deuteron as a whole), based oneself upon two experimental parameters — the deuteron binding energy B_d (13) and the Λ hyperon binding energy in the hypertriton B_Λ (14) — we find for the electric dipole polarizability of the hypertriton the value

$$\alpha_E^{\text{ZR,ZR}}(^3\Lambda\text{H}) = 1.701 \text{ fm}^3 . \quad (16)$$

Taking into account the finite value of the proton-neutron interaction radius through the application of the separable potential (8), whereas still describing the effective interaction between the Λ hyperon and the deuteron with the help of the zero-range interaction model, we obtain based oneself upon the three experimental

parameters — B_d , ${}^3a_{pn}$ (11) and B_Λ (12) — a larger magnitude of the hypertriton polarizability. For example, with the use of the Yukawa formfactor for the $n - p$ interaction potential and the zero-range model for the $\Lambda - d$ interaction, we find

$$\alpha_E^{\text{Yu,ZR}}({}^3_\Lambda\text{H}) = 2.479 \text{ fm}^3, \quad (17)$$

To evaluate the effect of the finite-range character of the effective $\Lambda - d$ interaction, it should be introduced one more parameter in addition. Taking into the consideration that no low-energy parameter characterizing the $\Lambda - d$ interaction other than B_Λ is experimentally determined, it is expedient to use the theoretical value of the doublet $\Lambda - d$ scattering length (15) as an additional parameter.

Tabulated in the Tables 2 and 3 are the calculated values of the electric dipole polarizabilities of the deuteron and the hypertriton, $\alpha_E({}^2\text{H})$ and $\alpha_E({}^3_\Lambda\text{H})$ (in fm^3), together with the geometrical characteristics of the nuclei (in fm) obtained with the use of the zero-range model and the separable potentials (10), (11) (with the formfactors (12) and the parameters given in the Table 1) for the description of the $p - n$ and $\Lambda - d$ interactions.

Table 2. The electric dipole polarizability of the deuteron $\alpha_E({}^2\text{H})$ (in fm^3) and the root-mean-square distances of the nucleons of the deuteron to its centre of mass, R_p and R_n , and between the nucleons of the deuteron, R_{pn} , (in fm) calculated for various potentials.

Quantity	$p - n$ interaction			
	ZR	Yu	Exp	DS
$\alpha_E({}^2\text{H})$	0.378	0.626	0.627	0.629
R_p	1.528	1.938	1.943	1.956
R_n	1.525	1.935	1.941	1.953
R_{pn}	3.053	3.873	3.884	3.908

Table 3. The electric dipole polarizability of the hypertriton $\alpha_E({}^3_\Lambda\text{H})$ (in fm^3) and the root-mean-square distances between each of the hadrons of the hypertriton (p, n, Λ) and the centre of mass of the hypertriton, R_p , R_n and R_Λ , and between the hadrons, R_{pn} , $R_{\Lambda p}$ and $R_{\Lambda n}$, (in fm) calculated for various potentials.

Quantity	$p - n, \Lambda - d$ interactions					
	ZR,ZR	Yu,ZR	ZR,Yu	Yu,Yu	Exp,Exp	DS,DS
$\alpha_E({}^3_\Lambda\text{H})$	1.701	2.479	2.002	2.914	2.933	2.964
R_p	4.148	4.315	4.470	4.626	4.629	4.636
R_n	4.147	4.314	4.469	4.625	4.628	4.635
R_Λ	6.490	4.490	7.070	7.070	7.072	7.074
R_{pn}	3.053	3.873	3.053	3.873	3.884	3.908
$R_{\Lambda p}$	10.458	10.526	11.374	11.436	11.439	11.446
$R_{\Lambda n}$	10.458	10.525	11.374	11.436	11.439	11.445

Note that in the zero-range interaction limit the hypertriton is found to be a more compact system than with the use of more realistic interaction which is characterized by a finite range. This is also evident from the calculated geometrical characteristics

of the system given in the Table 3. The more compact hypertriton system is certain to possess the lesser value of the electric polarizability.

For comparison, the electric dipole polarizability of the deuteron, calculated in the zero-range $p - n$ interaction limit, $\alpha_E^{\text{ZR}}(^2\text{H}) = 0.378 \text{ fm}^3$, is also decreased as against the values obtained with allowance for the finite magnitude of the range of the interaction, which are found to be $\alpha_E^{\text{Yu}}(^2\text{H}) = 0.626 \text{ fm}^3$ for the Yukawa formfactor, $\alpha_E^{\text{Exp}}(^2\text{H}) = 0.627 \text{ fm}^3$ for the exponential formfactor and $\alpha_E^{\text{DS}}(^2\text{H}) = 0.629 \text{ fm}^3$ for the delta-shell formfactor.

It follows from the results of our calculations presented in the Table 3 that taking into account the finite-range character of the $p - n$ interaction has a markedly more effect on the magnitude of the hypertriton electric polarizability $\alpha_E(^3_\Lambda\text{H})$ than that of the $\Lambda - d$ interaction: $\alpha_E^{\text{Yu,ZR}}(^3_\Lambda\text{H}) > \alpha_E^{\text{ZR,Yu}}(^3_\Lambda\text{H})$. This is due to the fact that the leading term of the effective range expansion (zero-range approximation) describes better the low-energy $\Lambda - d$ scattering as compared to the low-energy $p - n$ scattering because of the different extent of proximity to bound state pole in the corresponding transition matrices (the binding energy of the Λ hyperon in the hypertriton B_Λ is noticeably lesser than the deuteron binding energy B_d).

The calculated magnitude of the electric polarizability of the hypertriton depends only slightly on the kind of the used formfactor of the separable potential, varying from the value $\alpha_E^{\text{Yu,Yu}}(^3_\Lambda\text{H}) = 2.914 \text{ fm}^3$ for the Yukawa formfactor to the value $\alpha_E^{\text{DS,DS}}(^3_\Lambda\text{H}) = 2.964 \text{ fm}^3$ for the delta-shell one. Hence, from our results obtained it may be inferred that the value of the hypertriton polarizability $\alpha_E(^3_\Lambda\text{H})$ is close to 3 fm^3 exceeding the polarizabilities of the ordinary three-nucleon nuclei by an order of magnitude.

The Tables 2 and 3 list also geometrical characteristics of the deuteron and hypertriton, calculated with the use of the deuteron wave function $\psi_0(k)$ and the hypertriton wave function in the form (7): the root-mean-square distances between the corresponding particles and the centre of mass of the nuclei (R_p , R_n and R_Λ), together with the root-mean-square distances between the proton and the neutron (R_{pn}), between the Λ hyperon and the proton ($R_{\Lambda p}$) and between the Λ hyperon and the neutron ($R_{\Lambda n}$). Note that in the case of the application of the model wave function (7) the distance between the proton and the neutron in the hypertriton remains identical to that in the deuteron, $R_{pn}(^3_\Lambda\text{H}) = R_{pn}(^2\text{H})$, where $R_{pn}(^2\text{H}) = R_p(^2\text{H}) + R_n(^2\text{H})$. That there is only inconsiderable deformation of the deuteron in the hypertriton follows from the three-body calculations by Kolesnikov and Kalachev²⁸: $R_{pn}(^3_\Lambda\text{H})/R_{pn}(^2\text{H}) \approx 0.9$. According to our calculations, the root-mean-square distance between the Λ hyperon and the centre of mass of the hypertriton, R_Λ , is more than half as large again as the corresponding distances of the proton and the neutron, R_p and R_n . Hence, in the hypertriton the distances between the Λ hyperon and the proton and between the Λ hyperon and the neutron, $R_{\Lambda p}$ and $R_{\Lambda n}$, are more than twice as large as between the proton and neutron, R_{pn} .

5. Summary and conclusions

On the basis of the few-body approach, a rigorous formalism for determination of the electric dipole polarizability of a three-hadron bound complex that consists of one charged and two neutral particles and can form only one stable bound state has

been worked out. Leaning upon the analytical structure of the three-body transition matrix, a simple expression for the electric dipole polarizability of the hypertriton in terms of the partial derivatives of its wave function in momentum space has been formulated (Eq. (5)).

In the framework of the developed formalism, taking into consideration the specific structure of the hypertriton as a strange halo nucleus consisting of a tightly bound core (a ${}^2\text{H}$ nucleus) surrounded by the loosely bound Λ hyperon, the electric dipole polarizability of the hypertriton $\alpha_E({}^3_\Lambda\text{H})$ has been firstly calculated.

With the use of the $p - n$ separable interaction potential with the Yukawa formfactor and the $\Lambda - d$ zero-range interaction corresponding to the three experimentally established low-energy quantities — the binding energy of the deuteron B_d , the triplet $p - n$ scattering length ${}^3a_{pn}$ and the binding energy of the Λ hyperon in the hypertriton B_Λ — the value of the electric polarizability of the hypertriton is found to be $\alpha_E^{\text{Yu,ZR}}({}^3_\Lambda\text{H}) = 2.48 \text{ fm}^3$.

The evaluation of the finite-range effects of the $\Lambda - d$ interaction has been carried out using an additional theoretically calculated low-energy quantity — the doublet $\Lambda - d$ scattering length ${}^2a_{\Lambda d}$. It has been revealed that taking into account the finite-range character of the $\Lambda - d$ interaction causes a less pronounced increase of the hypertriton polarizability $\alpha_E({}^3_\Lambda\text{H})$ than that of the $p - n$ interaction. With the use of all the four $p - n$ and $\Lambda - d$ low-energy parameters, the calculated magnitude of the electric polarizability of the hypertriton $\alpha_E({}^3_\Lambda\text{H})$ depends only slightly on the kind of the formfactor of the $p - n$ and $\Lambda - d$ interaction potentials taking values close to 3 fm^3 (Table 3).

Thus it follows from our study that the electric polarizability of the Λ hypertriton ${}^3_\Lambda\text{H}$ by an order of magnitude exceeds the the polarizabilities of the ordinary three-nucleon systems — the triton and the helion-3. A further refinement of the hypertriton polarizability can be implemented with the use of three-body calculational techniques for obtaining the wave function of the bound state.

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